

CS103
WINTER 2025



Lecture 08:

Set Theory Revisited

Outline for Today

- ***Proofs on Sets***
 - Making our intuitions rigorous.
- ***Formal Set Definitions***
 - What do our terms mean?
- ***Appendices: Examples***
 - Sample proofs to help you get the hang of the ideas here.

Recap from Last Time

	If you <i>assume</i> this is true...	To <i>prove</i> that this is true...
$\forall x. A$	Initially, <i>do nothing</i> . Once you find a z through other means, you can state it has property A .	Have the reader pick an arbitrary x . We then prove A is true for that choice of x .
$\exists x. A$	Introduce a variable x into your proof that has property A .	Find an x where A is true. Then prove that A is true for that specific choice of x .
$A \rightarrow B$	Initially, <i>do nothing</i> . Once you know A is true, you can conclude B is also true.	Assume A is true, then prove B is true.
$A \wedge B$	Assume A . Also assume B .	Prove A . Also prove B .
$A \vee B$	Consider two cases. Case 1: A is true. Case 2: B is true.	Either prove $\neg A \rightarrow B$ or prove $\neg B \rightarrow A$. <i>(Why does this work?)</i>
$A \leftrightarrow B$	Assume $A \rightarrow B$ and $B \rightarrow A$.	Prove $A \rightarrow B$ and $B \rightarrow A$.
$\neg A$	Simplify the negation, then consult this table on the result.	Simplify the negation, then consult this table on the result.

New Stuff!

Proving Results from Set Theory

Claim: If A , B , and C are sets where $A \in B$ and $B \in C$, then $A \in C$.

Proof (?): Assume A , B , and C are sets where $A \in B$ and $B \in C$.

We need to show that $A \in C$.

Since $A \in B$, we know that A is contained in B . Since $B \in C$, we know that B is contained in C . Therefore, because A is contained in B and B is contained in C , we know that A is contained in C . This means that $A \in C$, as required. ■

Claim: If A , B , and C are sets where $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

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Which (if any) of these claims are true?
Answer at

<https://cs103.stanford.edu/pollev>

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This claim is not true! For example...

$$\begin{aligned}\emptyset &\in \{\emptyset\} \\ \{\emptyset\} &\in \{\{\emptyset\}\} \\ \emptyset &\notin \{\{\emptyset\}\}\end{aligned}$$

Lie: If A , B , and C are sets where $A \in B$ and $B \in C$, then $A \in C$.

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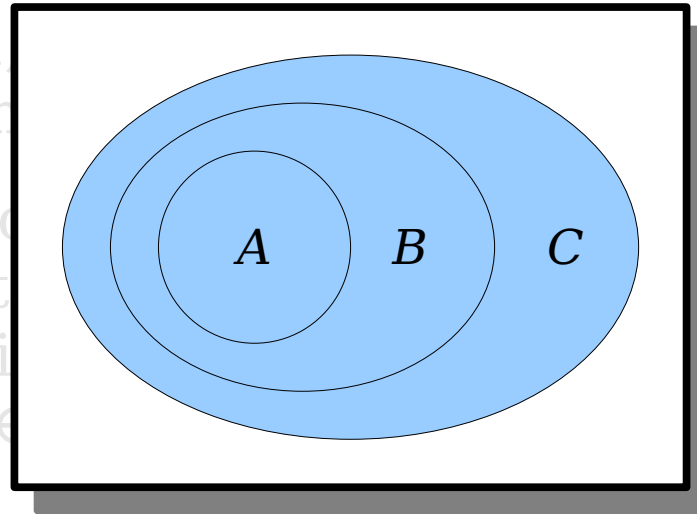
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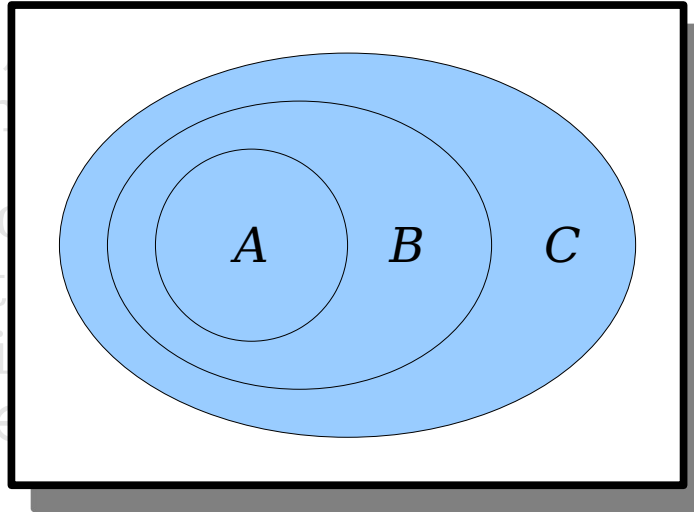
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This can't be a good proof;
the same basic argument
proves a false claim!

Claim: Let A , B , and C be sets. If $A \subseteq B$ and $A \subseteq C$, then we have $A \subseteq B \cap C$.

Proof (?): Assume $A \subseteq B$ and $A \subseteq C$. We need to show $A \subseteq B \cap C$.

Since $A \subseteq B$, all elements of A are in B . Since $A \subseteq C$, all elements of A are also in C . Therefore, all elements of A are in both B and C . Therefore, we see that $A \subseteq B \cap C$. ■

Claim: Let A , B , and C be sets. If $A \subsetneq B$ and $A \subsetneq C$, then we have $A \subsetneq B \cap C$.

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Which (if any) of these claims are true?
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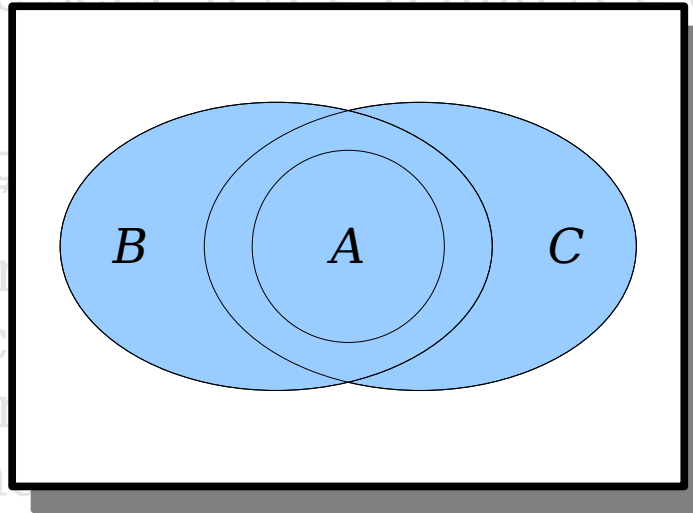
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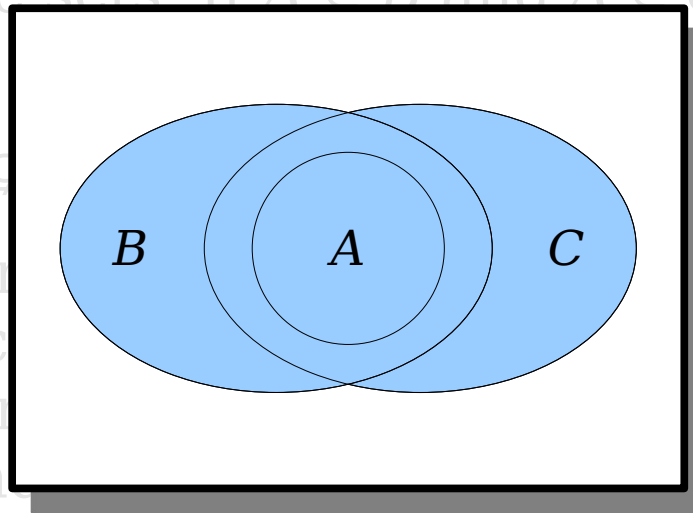
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What Went Wrong?

- The style of arguments you've just seen are **not** how to prove results on sets.
- As you've seen:
 - The reliance on high-level terms like “contained” is not mathematically precise.
 - A discussion of “all elements” of a set is not how to reason about collections of objects.
- **Question:** How do we write rigorous proofs about sets?

The Importance of Definitions

- As you saw last week, formal definitions underpin mathematical proofs.
- The major issue from the previous proofs is that we haven't defined what our terms mean.
 - How do we define what $A \in B$ means?
 - How do we define what $A \subseteq B$ means?
 - How do we define what $A \cap B$ means?
- Think back to our proof triangle: we currently have intuitions for these concepts, but not formal definitions.

	Is defined as...	If you <i>assume</i> this is true...	To <i>prove</i> that this is true...
$S \subseteq T$			
$S = T$			
$x \in S \cap T$			
$x \in S \cup T$			
$X \in \wp(S)$			
$x \in \{y \mid P(y)\}$			

Proofs on Subsets

Theorem: If A , B , and C are sets where
 $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

Defining Subsets

- Formally speaking, if S and T are sets, we say that $S \subseteq T$ when the following holds:

$$\forall x \in S. x \in T$$

- Now, suppose you're working with a proof where you encounter $S \subseteq T$. Think back to the proof table.
 - To **assume** that $S \subseteq T$, what should you do?
 - To **prove** that $S \subseteq T$, what should you do?

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Since $x \in A$ and $A \subseteq B$, we know $x \in B$. Similarly, since $x \in B$ and $B \subseteq C$, we see that $x \in C$, as required.

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Theorem: If A , B , and C are sets where $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

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Unions and Intersections

Theorem: Let A , B , and C be sets. If $A \subseteq B$ and $A \subseteq C$, then $A \subseteq B \cap C$.

Unions and Intersections

- The statement $x \in S \cap T$ is defined as

$$x \in S \quad \wedge \quad x \in T.$$

- The statement $x \in S \cup T$ is defined as

$$x \in S \quad \vee \quad x \in T.$$

- These are operational definitions: they show how unions and intersections interact with the \in relation rather than saying what the union or intersection of two sets “are.”

	Is defined as...	If you <i>assume</i> this is true...	To <i>prove</i> that this is true...
$S \subseteq T$	$\forall x \in S. x \in T$	Initially, <i>do nothing</i> . Once you find some $z \in S$, conclude $z \in T$.	Ask the reader to pick an $x \in S$. Then prove $x \in T$.
$S = T$			
$x \in S \cap T$	$x \in S \wedge x \in T$	Assume $x \in S$. Then assume $x \in T$.	Prove $x \in S$. Also prove $x \in T$.
$x \in S \cup T$	$x \in S \vee x \in T$	Consider two cases: Case 1: $x \in S$. Case 2: $x \in T$.	Either prove $x \in S$ or prove $x \in T$.
$X \in \wp(S)$			
$x \in \{y \mid P(y)\}$			

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	Is defined as...	If you <i>assume</i> this is true...	To <i>prove</i> that this is true...
$S \subseteq T$	$\forall x \in S. x \in T$	Initially, <i>do nothing</i> . Once you find some $z \in S$, conclude $z \in T$.	Ask the reader to pick an $x \in S$. Then prove $x \in T$

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$S \subseteq T$	$\forall x \in S. x \in T$	Initially, <i>do nothing</i> . Once you find some $z \in S$, conclude $z \in T$.	Ask the reader to pick an $x \in S$. Then prove $x \in T$

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Since $x \in A$ and $A \subseteq B$, we know $x \in B$.

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Bad Proof: Assume $A \subseteq B$ and $A \subseteq C$. We need to show $A \subseteq B \cap C$.

Since $A \subseteq B$, all elements of A are in B . Since $A \subseteq C$, all elements of A are also in C . Therefore, all elements of A are in both B and C . Therefore, we see that $A \subseteq B \cap C$. ■

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Set Equality

Theorem: Let A and B be sets. Then $A \subseteq B$ if and only if $A \cup B = B$.

	Is defined as...	If you <i>assume</i> this is true...	To <i>prove</i> that this is true...
$S \subseteq T$	$\forall x \in S. x \in T$	Initially, <i>do nothing</i> . Once you find some $z \in S$, conclude $z \in T$.	Ask the reader to pick an $x \in S$. Then prove $x \in T$.
$S = T$	$S \subseteq T \wedge T \subseteq S$	Assume $S \subseteq T$ and $T \subseteq S$.	Prove $S \subseteq T$. Also prove $T \subseteq S$.
$x \in S \cap T$	$x \in S \wedge x \in T$	Assume $x \in S$. Then assume $x \in T$.	Prove $x \in S$. Also prove $x \in T$.
$x \in S \cup T$	$x \in S \vee x \in T$	Consider two cases: Case 1: $x \in S$. Case 2: $x \in T$.	Either prove $x \in S$ or prove $x \in T$.
$X \in \wp(S)$			
$x \in \{y \mid P(y)\}$			

Theorem: Let A and B be sets. Then $A \subseteq B$ if and only if $A \cup B = B$.

Proof: See appendix!

Set-Builder Notation

Set-Builder Notation

- Let S be the set defined here:

$$S = \{ n \mid n \in \mathbb{N} \text{ and } n \geq 137 \}$$

- Now imagine you have some quantity x . Based on this...
 - ... if you **assume** that $x \in S$, what does that tell you about x ?
 - ... if you need to **prove** that $x \in S$, what do you need to prove?

Answer at

<https://cs103.stanford.edu/pollev>

Set-Builder Notation

- Like unions and intersections, we have an operational definition for set-builder notation. It's the following:

Let $S = \{ y \mid P(y) \}$.
Then $x \in S$ when $P(x)$ is true.

- So, for example:
 - $x \in \{ n \mid n \in \mathbb{N} \text{ and } n \text{ is even} \}$ means $x \in \mathbb{N}$ and x is even.
 - $x \in \{ n \mid \exists k \in \mathbb{N}. n = 2k + 1 \}$ means that there is a $k \in \mathbb{N}$ where $x = 2k + 1$. (Equivalently, x is an odd natural number)
- **Key Point:** The placeholder variable disappears in all these examples. After all, *it's just a placeholder*.

	Is defined as...	If you <i>assume</i> this is true...	To <i>prove</i> that this is true...
$S \subseteq T$	$\forall x \in S. x \in T$	Initially, <i>do nothing</i> . Once you find some $z \in S$, conclude $z \in T$.	Ask the reader to pick an $x \in S$. Then prove $x \in T$.
$S = T$	$S \subseteq T \wedge T \subseteq S$	Assume $S \subseteq T$ and $T \subseteq S$.	Prove $S \subseteq T$. Also prove $T \subseteq S$.
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$X \in \wp(S)$			
$x \in \{y \mid P(y)\}$	$P(x)$	Assume $P(x)$.	Prove $P(x)$.

Proofs on Set-Builder Notation

Some Useful Notation

- If n is a natural number, we define the set **$[n]$** as follows:

$$[n] = \{ k \mid k \in \mathbb{N} \wedge k < n \}$$

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- If n is a natural number, we define the set **$[n]$** as follows:

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- So, for example:
 - $[3] =$

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- If n is a natural number, we define the set **$[n]$** as follows:

$$[n] = \{ k \mid k \in \mathbb{N} \wedge k < n \}$$

- So, for example:
 - $[3] = \{0, 1, 2\}$
 - $[0] =$

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- If n is a natural number, we define the set **$[n]$** as follows:

$$[n] = \{ k \mid k \in \mathbb{N} \wedge k < n \}$$

- So, for example:
 - $[3] = \{0, 1, 2\}$
 - $[0] = \emptyset$
 - $[5] =$

Some Useful Notation

- If n is a natural number, we define the set **$[n]$** as follows:

$$[n] = \{ k \mid k \in \mathbb{N} \wedge k < n \}$$

- So, for example:
 - $[3] = \{0, 1, 2\}$
 - $[0] = \emptyset$
 - $[5] = \{0, 1, 2, 3, 4\}$

Theorem: If $m, n \in \mathbb{N}$ and $m < n$,
then $[m] \subseteq [n]$

Theorem: If $m, n \in \mathbb{N}$ and $m < n$, then $[m] \subseteq [n]$

What We're Assuming

$$m \in \mathbb{N}$$

$$n \in \mathbb{N}$$

$$m < n$$

$$[z] = \{ k \mid k \in \mathbb{N} \wedge k < z \}$$

What We Need to Prove

$$[m] \subseteq [n]$$

Theorem: If $m, n \in \mathbb{N}$ and $m < n$, then $[m] \subseteq [n]$

What We're Assuming

$$m \in \mathbb{N}$$

$$n \in \mathbb{N}$$

$$m < n$$

$$[z] = \{ k \mid k \in \mathbb{N} \wedge k < z \}$$

What We Need to Prove

$$\cancel{[m] \subseteq [n]}$$

$$\forall x \in [m]. x \in [n]$$

Theorem: If $m, n \in \mathbb{N}$ and $m < n$, then $[m] \subseteq [n]$

What We're Assuming

$$m \in \mathbb{N}$$

$$n \in \mathbb{N}$$

$$m < n$$

$$[z] = \{ k \mid k \in \mathbb{N} \wedge k < z \}$$

$$x \in [m]$$

What We Need to Prove

$$\cancel{[m] \subseteq [n]}$$

$$\forall x \in [m]. x \in [n]$$

Theorem: If $m, n \in \mathbb{N}$ and $m < n$, then $[m] \subseteq [n]$

What We're Assuming

$$m \in \mathbb{N}$$

$$n \in \mathbb{N}$$

$$m < n$$

$$[z] = \{ k \mid k \in \mathbb{N} \wedge k < z \}$$

$$x \in [m]$$

$$x \in \mathbb{N}$$

$$x < m$$

What We Need to Prove

$$\cancel{[m] \subseteq [n]}$$

$$\forall x \in \cancel{[m]}. x \in [n]$$

Theorem: If $m, n \in \mathbb{N}$ and $m < n$, then $[m] \subseteq [n]$

What We're Assuming

$$m \in \mathbb{N}$$

$$n \in \mathbb{N}$$

$$m < n$$

$$[z] = \{ k \mid k \in \mathbb{N} \wedge k < z \}$$

$$x \in [m]$$

$$x \in \mathbb{N}$$

$$x < m$$

What We Need to Prove

$$\cancel{[m] \subseteq [n]}$$

$$\cancel{\forall x \in [m]. x \in [n]}$$

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Proof: Assume m and n are natural numbers where $m < n$. We need to show that $[m] \subseteq [n]$. To do so, pick some $x \in [m]$.

Theorem: If $m, n \in \mathbb{N}$ and $m < n$, then $[m] \subseteq [n]$.

Proof: Assume m and n are natural numbers where $m < n$. We need to show that $[m] \subseteq [n]$. To do so, pick some $x \in [m]$. We'll prove that $x \in [n]$.

Theorem: If $m, n \in \mathbb{N}$ and $m < n$, then $[m] \subseteq [n]$.

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Since $x \in [m]$, we know that $x \in \mathbb{N}$ and $x < m$.

Theorem: If $m, n \in \mathbb{N}$ and $m < n$, then $[m] \subseteq [n]$.

Proof: Assume m and n are natural numbers where $m < n$. We need to show that $[m] \subseteq [n]$. To do so, pick some $x \in [m]$. We'll prove that $x \in [n]$.

Since $x \in [m]$, we know that $x \in \mathbb{N}$ and $x < m$. Then, because $x < m$ and $m < n$, we know that $x < n$.

Theorem: If $m, n \in \mathbb{N}$ and $m < n$, then $[m] \subseteq [n]$.

Proof: Assume m and n are natural numbers where $m < n$. We need to show that $[m] \subseteq [n]$. To do so, pick some $x \in [m]$. We'll prove that $x \in [n]$.

Since $x \in [m]$, we know that $x \in \mathbb{N}$ and $x < m$. Then, because $x < m$ and $m < n$, we know that $x < n$.

Collectively this means that $x \in \mathbb{N}$ and $x < n$, so $x \in [n]$, as required.

Theorem: If $m, n \in \mathbb{N}$ and $m < n$, then $[m] \subseteq [n]$.

Proof: Assume m and n are natural numbers where $m < n$. We need to show that $[m] \subseteq [n]$. To do so, pick some $x \in [m]$. We'll prove that $x \in [n]$.

Since $x \in [m]$, we know that $x \in \mathbb{N}$ and $x < m$. Then, because $x < m$ and $m < n$, we know that $x < n$.

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Since $x \in [m]$, we know that $x \in \mathbb{N}$ and $x < m$. Then, because $x < m$ and $m < n$, we know that $x < n$.

Collectively this means that $x \in \mathbb{N}$ and $x < n$, so $x \in [n]$, as required. ■

Notice that *there is no set-builder notation in this proof*. We were able to avoid it by using the rules for what $x \in \{y \mid P(y)\}$ say to do.

	Is defined as...	If you <i>assume</i> this is true...	To <i>prove</i> that this is true...
$S \subseteq T$	$\forall x \in S. x \in T$	Initially, <i>do nothing</i> . Once you find some $z \in S$, conclude $z \in T$.	Ask the reader to pick an $x \in S$. Then prove $x \in T$.
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$X \in \wp(S)$	$X \subseteq S$.	Assume $X \subseteq S$.	Prove $X \subseteq S$.
$x \in \{y \mid P(y)\}$	$P(x)$	Assume $P(x)$.	Prove $P(x)$.

Announcements

- ***Upcoming Deadlines***
 - (**Today**, 11:59 PM): Left-handed exam desk requests
 - (**Friday**, 1:00 PM): Problem Set 3
 - Start early and make slow and steady progress.
 - Attend OH sooner rather than later.
 - (**Friday**, 11:59 PM): Attendance opt-out
 - (**Sunday**, 7 PM): Midterm 1 Bonus

Announcements

- ***Hodgepodge***

- Problem Set 3 LaTeX template updated Sunday evening.
- Task map updated.
- New “Resources” for the week now tagged as such in drop-down on course website.
- Looking for a pset partner? Post on Ed!
- Sean’s OH have moved to CoDa.

- ***Midterm 1***

- Logistics page unlocked.
- So many practice problems.
- More on Wednesday.

Readings

- ***Read “Guide to Proofs on Discrete Structures.”***
 - There’s additional guidance and practice on the assume/prove dichotomy and how it manifests in problem-solving.
- ***Read “Discrete Structures Proofwriting Checklist.”***
 - Keep the items here in mind when writing proofs. We’ll use this when grading your problem set.
- ***Read “Guide to Proofs on Sets.”***
 - There’s some good worked examples in there to supplement today’s lecture, several of which will be relevant for the problem set.

Next Time

- ***Graph Theory***

- A ubiquitous, powerful abstraction with applications throughout computer science.

- ***Vertex Covers***

- Making sure tourists don't get lost.

- ***Independent Sets***

- Helping the recovery of the California Condor.

Appendix: More Sample Set Proofs

Theorem: Let A , B , C , and D be sets where
 $A \subseteq C$ and $B \subseteq D$. Then $A \cup B \subseteq C \cup D$.

Theorem: Let A , B , C , and D be sets where
 $A \subseteq C$ and $B \subseteq D$. Then $A \cup B \subseteq C \cup D$.

What I'm Assuming

$$A \subseteq C$$

$$\forall z \in A. z \in C$$

$$B \subseteq D$$

$$\forall z \in B. z \in D$$

What I Need to Show

$$A \cup B \subseteq C \cup D$$

Theorem: Let A , B , C , and D be sets where
 $A \subseteq C$ and $B \subseteq D$. Then $A \cup B \subseteq C \cup D$.

What I'm Assuming

$$A \subseteq C$$

$$\forall z \in A. z \in C$$

$$B \subseteq D$$

$$\forall z \in B. z \in D$$

What I Need to Show

$$\cancel{A \cup B \subseteq C \cup D}$$

$$\forall x \in A \cup B. x \in C \cup D$$

Theorem: Let A , B , C , and D be sets where
 $A \subseteq C$ and $B \subseteq D$. Then $A \cup B \subseteq C \cup D$.

What I'm Assuming

$$A \subseteq C$$

$$\forall z \in A. z \in C$$

$$B \subseteq D$$

$$\forall z \in B. z \in D$$

$$x \in A \cup B$$

What I Need to Show

~~$$A \cup B \subseteq C \cup D$$~~

~~$$\forall x \in A \cup B. x \in C \cup D$$~~

Theorem: Let A , B , C , and D be sets where
 $A \subseteq C$ and $B \subseteq D$. Then $A \cup B \subseteq C \cup D$.

What I'm Assuming

$$A \subseteq C$$

$$\forall z \in A. z \in C$$

$$B \subseteq D$$

$$\forall z \in B. z \in D$$

$$x \in A \cup B$$

What I Need to Show

~~$$A \cup B \subseteq C \cup D$$~~

~~$$\forall x \in A \cup B. x \in C \cup D$$~~

$$x \in C \text{ or } x \in D$$

Theorem: Let A , B , C , and D be sets where
 $A \subseteq C$ and $B \subseteq D$. Then $A \cup B \subseteq C \cup D$.

What I'm Assuming

$$A \subseteq C$$

$$\forall z \in A. z \in C$$

$$B \subseteq D$$

$$\forall z \in B. z \in D$$

$$x \in A \cup B$$

Case 1: $x \in A$

Case 2: $x \in B$

What I Need to Show

~~$$A \cup B \subseteq C \cup D$$~~

~~$$\forall x \in A \cup B. x \in C \cup D$$~~

$$x \in C \text{ or } x \in D$$

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Proof:

Theorem: Let A , B , C , and D be sets where $A \subseteq C$ and $B \subseteq D$. Then $A \cup B \subseteq C \cup D$.

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Theorem: Let A , B , C , and D be sets where $A \subseteq C$ and $B \subseteq D$. Then $A \cup B \subseteq C \cup D$.

Proof: Pick some $x \in A \cup B$; we need to show that $x \in C \cup D$.

Because $x \in A \cup B$, we know that $x \in A$ or $x \in B$.

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Because $x \in A \cup B$, we know that $x \in A$ or $x \in B$. We consider each case separately.

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Proof: Pick some $x \in A \cup B$; we need to show that $x \in C \cup D$.

Because $x \in A \cup B$, we know that $x \in A$ or $x \in B$. We consider each case separately.

Case 1: $x \in A$.

Case 2: $x \in B$.

Theorem: Let A , B , C , and D be sets where $A \subseteq C$ and $B \subseteq D$. Then $A \cup B \subseteq C \cup D$.

Proof: Pick some $x \in A \cup B$; we need to show that $x \in C \cup D$.

Because $x \in A \cup B$, we know that $x \in A$ or $x \in B$. We consider each case separately.

Case 1: $x \in A$. Since $A \subseteq C$ and $x \in A$, we see that $x \in C$, and therefore that $x \in C \cup D$.

Case 2: $x \in B$.

Theorem: Let A , B , C , and D be sets where $A \subseteq C$ and $B \subseteq D$. Then $A \cup B \subseteq C \cup D$.

Proof: Pick some $x \in A \cup B$; we need to show that $x \in C \cup D$.

Because $x \in A \cup B$, we know that $x \in A$ or $x \in B$. We consider each case separately.

Case 1: $x \in A$. Since $A \subseteq C$ and $x \in A$, we see that $x \in C$, and therefore that $x \in C \cup D$.

Case 2: $x \in B$. Then because $B \subseteq D$ and $x \in B$ we have $x \in D$, so $x \in C \cup D$.

Theorem: Let A , B , C , and D be sets where $A \subseteq C$ and $B \subseteq D$. Then $A \cup B \subseteq C \cup D$.

Proof: Pick some $x \in A \cup B$; we need to show that $x \in C \cup D$.

Because $x \in A \cup B$, we know that $x \in A$ or $x \in B$. We consider each case separately.

Case 1: $x \in A$. Since $A \subseteq C$ and $x \in A$, we see that $x \in C$, and therefore that $x \in C \cup D$.

Case 2: $x \in B$. Then because $B \subseteq D$ and $x \in B$ we have $x \in D$, so $x \in C \cup D$.

In either case, we see that $x \in C \cup D$, as required.

Theorem: Let A , B , C , and D be sets where $A \subseteq C$ and $B \subseteq D$. Then $A \cup B \subseteq C \cup D$.

Proof: Pick some $x \in A \cup B$; we need to show that $x \in C \cup D$.

Because $x \in A \cup B$, we know that $x \in A$ or $x \in B$. We consider each case separately.

Case 1: $x \in A$. Since $A \subseteq C$ and $x \in A$, we see that $x \in C$, and therefore that $x \in C \cup D$.

Case 2: $x \in B$. Then because $B \subseteq D$ and $x \in B$ we have $x \in D$, so $x \in C \cup D$.

In either case, we see that $x \in C \cup D$, as required. ■

Theorem: Let A and B be sets. Then $A \subseteq B$ if and only if $A \cup B = B$.

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Theorem: Let A and B be sets. Then $A \subseteq B$ if and only if $A \cup B = B$.

Proof: We will prove each direction of implication.

(\Rightarrow) Assume $A \subseteq B$. We need to show that $A \cup B = B$. To do so, we need to show that $A \cup B \subseteq B$ and that $B \subseteq A \cup B$.

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First, we'll show $A \cup B \subseteq B$.

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First, we'll show $A \cup B \subseteq B$. Pick an $x \in A \cup B$.

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First, we'll show $A \cup B \subseteq B$. Pick an $x \in A \cup B$. We need to show that $x \in B$. Since $x \in A \cup B$, we consider two cases:

Case 1: $x \in A$.

Case 2: $x \in B$.

(\Leftarrow) Assume $A \cup B = B$. We need to show that $A \subseteq B$.

Theorem: Let A and B be sets. Then $A \subseteq B$ if and only if $A \cup B = B$.

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Case 1: $x \in A$. Then since $x \in A$ and $A \subseteq B$, we have $x \in B$.

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Theorem: Let A and B be sets. Then $A \subseteq B$ if and only if $A \cup B = B$.

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Case 1: $x \in A$. Then since $x \in A$ and $A \subseteq B$, we have $x \in B$.

Case 2: $x \in B$. Then by assumption $x \in B$.

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First, we'll show $A \cup B \subseteq B$. Pick an $x \in A \cup B$. We need to show that $x \in B$. Since $x \in A \cup B$, we consider two cases:

Case 1: $x \in A$. Then since $x \in A$ and $A \subseteq B$, we have $x \in B$.

Case 2: $x \in B$. Then by assumption $x \in B$.

Either way, we have $x \in B$, which is what we needed to show.

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Theorem: Let A and B be sets. Then $A \subseteq B$ if and only if $A \cup B = B$.

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(\Rightarrow) Assume $A \subseteq B$. We need to show that $A \cup B = B$. To do so, we need to show that $A \cup B \subseteq B$ and that $B \subseteq A \cup B$.

First, we'll show $A \cup B \subseteq B$. Pick an $x \in A \cup B$. We need to show that $x \in B$. Since $x \in A \cup B$, we consider two cases:

Case 1: $x \in A$. Then since $x \in A$ and $A \subseteq B$, we have $x \in B$.

Case 2: $x \in B$. Then by assumption $x \in B$.

Either way, we have $x \in B$, which is what we needed to show.

Next, we'll prove $B \subseteq A \cup B$.

(\Leftarrow) Assume $A \cup B = B$. We need to show that $A \subseteq B$.

Theorem: Let A and B be sets. Then $A \subseteq B$ if and only if $A \cup B = B$.

Proof: We will prove each direction of implication.

(\Rightarrow) Assume $A \subseteq B$. We need to show that $A \cup B = B$. To do so, we need to show that $A \cup B \subseteq B$ and that $B \subseteq A \cup B$.

First, we'll show $A \cup B \subseteq B$. Pick an $x \in A \cup B$. We need to show that $x \in B$. Since $x \in A \cup B$, we consider two cases:

Case 1: $x \in A$. Then since $x \in A$ and $A \subseteq B$, we have $x \in B$.

Case 2: $x \in B$. Then by assumption $x \in B$.

Either way, we have $x \in B$, which is what we needed to show.

Next, we'll prove $B \subseteq A \cup B$. Pick some $x \in B$.

(\Leftarrow) Assume $A \cup B = B$. We need to show that $A \subseteq B$.

Theorem: Let A and B be sets. Then $A \subseteq B$ if and only if $A \cup B = B$.

Proof: We will prove each direction of implication.

(\Rightarrow) Assume $A \subseteq B$. We need to show that $A \cup B = B$. To do so, we need to show that $A \cup B \subseteq B$ and that $B \subseteq A \cup B$.

First, we'll show $A \cup B \subseteq B$. Pick an $x \in A \cup B$. We need to show that $x \in B$. Since $x \in A \cup B$, we consider two cases:

Case 1: $x \in A$. Then since $x \in A$ and $A \subseteq B$, we have $x \in B$.

Case 2: $x \in B$. Then by assumption $x \in B$.

Either way, we have $x \in B$, which is what we needed to show.

Next, we'll prove $B \subseteq A \cup B$. Pick some $x \in B$. Since $x \in B$, we know that $x \in A \cup B$, as required.

(\Leftarrow) Assume $A \cup B = B$. We need to show that $A \subseteq B$.

Theorem: Let A and B be sets. Then $A \subseteq B$ if and only if $A \cup B = B$.

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(\Rightarrow) Assume $A \subseteq B$. We need to show that $A \cup B = B$. To do so, we need to show that $A \cup B \subseteq B$ and that $B \subseteq A \cup B$.

First, we'll show $A \cup B \subseteq B$. Pick an $x \in A \cup B$. We need to show that $x \in B$. Since $x \in A \cup B$, we consider two cases:

Case 1: $x \in A$. Then since $x \in A$ and $A \subseteq B$, we have $x \in B$.

Case 2: $x \in B$. Then by assumption $x \in B$.

Either way, we have $x \in B$, which is what we needed to show.

Next, we'll prove $B \subseteq A \cup B$. Pick some $x \in B$. Since $x \in B$, we know that $x \in A \cup B$, as required.

(\Leftarrow) Assume $A \cup B = B$. We need to show that $A \subseteq B$. So pick an $x \in A$; we need to show that $x \in B$.

Theorem: Let A and B be sets. Then $A \subseteq B$ if and only if $A \cup B = B$.

Proof: We will prove each direction of implication.

(\Rightarrow) Assume $A \subseteq B$. We need to show that $A \cup B = B$. To do so, we need to show that $A \cup B \subseteq B$ and that $B \subseteq A \cup B$.

First, we'll show $A \cup B \subseteq B$. Pick an $x \in A \cup B$. We need to show that $x \in B$. Since $x \in A \cup B$, we consider two cases:

Case 1: $x \in A$. Then since $x \in A$ and $A \subseteq B$, we have $x \in B$.

Case 2: $x \in B$. Then by assumption $x \in B$.

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Next, we'll prove $B \subseteq A \cup B$. Pick some $x \in B$. Since $x \in B$, we know that $x \in A \cup B$, as required.

(\Leftarrow) Assume $A \cup B = B$. We need to show that $A \subseteq B$. So pick an $x \in A$; we need to show that $x \in B$.

Since $x \in A$, we know that $x \in A \cup B$.

Theorem: Let A and B be sets. Then $A \subseteq B$ if and only if $A \cup B = B$.

Proof: We will prove each direction of implication.

(\Rightarrow) Assume $A \subseteq B$. We need to show that $A \cup B = B$. To do so, we need to show that $A \cup B \subseteq B$ and that $B \subseteq A \cup B$.

First, we'll show $A \cup B \subseteq B$. Pick an $x \in A \cup B$. We need to show that $x \in B$. Since $x \in A \cup B$, we consider two cases:

Case 1: $x \in A$. Then since $x \in A$ and $A \subseteq B$, we have $x \in B$.

Case 2: $x \in B$. Then by assumption $x \in B$.

Either way, we have $x \in B$, which is what we needed to show.

Next, we'll prove $B \subseteq A \cup B$. Pick some $x \in B$. Since $x \in B$, we know that $x \in A \cup B$, as required.

(\Leftarrow) Assume $A \cup B = B$. We need to show that $A \subseteq B$. So pick an $x \in A$; we need to show that $x \in B$.

Since $x \in A$, we know that $x \in A \cup B$. And since $x \in A \cup B$ and $A \cup B = B$, we see that $x \in B$, as required.

Theorem: Let A and B be sets. Then $A \subseteq B$ if and only if $A \cup B = B$.

Proof: We will prove each direction of implication.

(\Rightarrow) Assume $A \subseteq B$. We need to show that $A \cup B = B$. To do so, we need to show that $A \cup B \subseteq B$ and that $B \subseteq A \cup B$.

First, we'll show $A \cup B \subseteq B$. Pick an $x \in A \cup B$. We need to show that $x \in B$. Since $x \in A \cup B$, we consider two cases:

Case 1: $x \in A$. Then since $x \in A$ and $A \subseteq B$, we have $x \in B$.

Case 2: $x \in B$. Then by assumption $x \in B$.

Either way, we have $x \in B$, which is what we needed to show.

Next, we'll prove $B \subseteq A \cup B$. Pick some $x \in B$. Since $x \in B$, we know that $x \in A \cup B$, as required.

(\Leftarrow) Assume $A \cup B = B$. We need to show that $A \subseteq B$. So pick an $x \in A$; we need to show that $x \in B$.

Since $x \in A$, we know that $x \in A \cup B$. And since $x \in A \cup B$ and $A \cup B = B$, we see that $x \in B$, as required. ■

Theorem: Let A and B be sets. Then if $\wp(A) = \wp(B)$, then $A = B$.

Theorem: Let A and B be sets. Then
if $\wp(A) = \wp(B)$, then $A = B$.

What I'm Assuming

$$\wp(A) = \wp(B)$$

$$\wp(A) \subseteq \wp(B)$$

$$\forall Z \in \wp(A). Z \in \wp(B)$$

$$\wp(B) \subseteq \wp(A)$$

$$\forall Z \in \wp(B). Z \in \wp(A)$$

What I Need to Show

$$A = B$$

Theorem: Let A and B be sets. Then
if $\wp(A) = \wp(B)$, then $A = B$.

What I'm Assuming

$$\wp(A) = \wp(B)$$

$$\wp(A) \subseteq \wp(B)$$

$$\forall Z \in \wp(A). Z \in \wp(B)$$

$$\wp(B) \subseteq \wp(A)$$

$$\forall Z \in \wp(B). Z \in \wp(A)$$

What I Need to Show

$$\cancel{A = B}$$

$$A \subseteq B$$

$$B \subseteq A$$

Theorem: Let A and B be sets. Then
if $\wp(A) = \wp(B)$, then $A = B$.

What I'm Assuming

$$\wp(A) = \wp(B)$$

$$\wp(A) \subseteq \wp(B)$$

$$\forall Z \in \wp(A). Z \in \wp(B)$$

$$\wp(B) \subseteq \wp(A)$$

$$\forall Z \in \wp(B). Z \in \wp(A)$$

What I Need to Show

$$\cancel{A = B}$$

$$\cancel{A \subseteq B}$$

$$\forall x \in A. x \in B.$$

$$\cancel{B \subseteq A}$$

$$\forall z \in B. z \in A.$$

Theorem: Let A and B be sets. Then
if $\wp(A) = \wp(B)$, then $A = B$.

What I'm Assuming

$$\wp(A) = \wp(B)$$

$$\wp(A) \subseteq \wp(B)$$

$$\forall Z \in \wp(A). Z \in \wp(B)$$

$$\wp(B) \subseteq \wp(A)$$

$$\forall Z \in \wp(B). Z \in \wp(A)$$

$$x \in A$$

$$\{x\} \subseteq A$$

$$\{x\} \in \wp(A)$$

What I Need to Show

~~$$A = B$$~~

~~$$A \subseteq B$$~~

~~$$\forall x \in A. x \in B.$$~~

~~$$B \subseteq A$$~~

$$\forall z \in B. z \in A.$$

Theorem: Let A and B be sets. Then
if $\wp(A) = \wp(B)$, then $A = B$.

What I'm Assuming

$$\wp(A) = \wp(B)$$

$$\wp(A) \subseteq \wp(B)$$

$$\forall Z \in \wp(A). Z \in \wp(B)$$

$$\wp(B) \subseteq \wp(A)$$

$$\forall Z \in \wp(B). Z \in \wp(A)$$

$$x \in A$$

$$x \in B$$

$$\{x\} \subseteq A$$

$$\{x\} \subseteq B$$

$$\{x\} \in \wp(A)$$

$$\{x\} \in \wp(B)$$

What I Need to Show

~~$$A = B$$~~

~~$$A \subseteq B$$~~

~~$$\forall x \in A. x \in B.$$~~

~~$$B \subseteq A$$~~

~~$$\forall z \in B. z \in A.$$~~

Theorem: Let A and B be sets. If $\wp(A) = \wp(B)$,
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Proof:

Theorem: Let A and B be sets. If $\wp(A) = \wp(B)$, then $A = B$.

Proof: Assume $\wp(A) = \wp(B)$.

Theorem: Let A and B be sets. If $\wp(A) = \wp(B)$, then $A = B$.

Proof: Assume $\wp(A) = \wp(B)$. We need to show that $A = B$, or, equivalently, that $A \subseteq B$ and $B \subseteq A$.

Theorem: Let A and B be sets. If $\wp(A) = \wp(B)$, then $A = B$.

Proof: Assume $\wp(A) = \wp(B)$. We need to show that $A = B$, or, equivalently, that $A \subseteq B$ and $B \subseteq A$. Since the roles of A and B are symmetric, we'll just prove $A \subseteq B$.

Theorem: Let A and B be sets. If $\wp(A) = \wp(B)$, then $A = B$.

Proof: Assume $\wp(A) = \wp(B)$. We need to show that $A = B$, or, equivalently, that $A \subseteq B$ and $B \subseteq A$. Since the roles of A and B are symmetric, we'll just prove $A \subseteq B$.

Pick some $x \in A$; we need to show that $x \in B$.

Theorem: Let A and B be sets. If $\wp(A) = \wp(B)$, then $A = B$.

Proof: Assume $\wp(A) = \wp(B)$. We need to show that $A = B$, or, equivalently, that $A \subseteq B$ and $B \subseteq A$. Since the roles of A and B are symmetric, we'll just prove $A \subseteq B$.

Pick some $x \in A$; we need to show that $x \in B$. Since $x \in A$, we know that $\{x\} \subseteq A$.

Theorem: Let A and B be sets. If $\wp(A) = \wp(B)$, then $A = B$.

Proof: Assume $\wp(A) = \wp(B)$. We need to show that $A = B$, or, equivalently, that $A \subseteq B$ and $B \subseteq A$. Since the roles of A and B are symmetric, we'll just prove $A \subseteq B$.

Pick some $x \in A$; we need to show that $x \in B$. Since $x \in A$, we know that $\{x\} \subseteq A$. This means that $\{x\} \in \wp(A)$, and since $\wp(A) \subseteq \wp(B)$ we know $\{x\} \in \wp(B)$.

Theorem: Let A and B be sets. If $\wp(A) = \wp(B)$, then $A = B$.

Proof: Assume $\wp(A) = \wp(B)$. We need to show that $A = B$, or, equivalently, that $A \subseteq B$ and $B \subseteq A$. Since the roles of A and B are symmetric, we'll just prove $A \subseteq B$.

Pick some $x \in A$; we need to show that $x \in B$. Since $x \in A$, we know that $\{x\} \subseteq A$. This means that $\{x\} \in \wp(A)$, and since $\wp(A) \subseteq \wp(B)$ we know $\{x\} \in \wp(B)$. Thus we see that $\{x\} \subseteq B$, which in turn means that $x \in B$, as required.

Theorem: Let A and B be sets. If $\wp(A) = \wp(B)$, then $A = B$.

Proof: Assume $\wp(A) = \wp(B)$. We need to show that $A = B$, or, equivalently, that $A \subseteq B$ and $B \subseteq A$. Since the roles of A and B are symmetric, we'll just prove $A \subseteq B$.

Pick some $x \in A$; we need to show that $x \in B$. Since $x \in A$, we know that $\{x\} \subseteq A$. This means that $\{x\} \in \wp(A)$, and since $\wp(A) \subseteq \wp(B)$ we know $\{x\} \in \wp(B)$. Thus we see that $\{x\} \subseteq B$, which in turn means that $x \in B$, as required. ■